

The Pinch Technique at Two Loops

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It is shown that the fundamental properties of gauge-independence, gauge-invariance, unitarity, and analyticity of the S -matrix lead to the unambiguous generalization of the pinch technique algorithm to two loops.

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A variety of important physical problems cannot be addressed within the framework of fixed-order perturbation theory, the most widely used calculational scheme in the continuum. This is often the case within Quantum Chromodynamics (QCD), when large disparities in the physical scales involved result in a complicated interplay between perturbative and non-perturbative effects. Similar limitations appear when physical kinematic singularities, such as resonances, render the perturbative expansion divergent at any finite order, or when perturbatively exact symmetries prohibit the appearance of certain phenomena, such as chiral symmetry breaking or gluon mass generation. In such cases one often resorts to various reorganizations of the perturbative expansion inspired from scalar field theories or Quantum Electrodynamics (QED), supplemented by a number of auxiliary physical principles. When studying the interface between perturbative and non-perturbative QCD for example, one finds it advantageous to use concepts familiar from QED, such as the effective charge, in conjunction with dispersive techniques and analyticity properties of the S -matrix [1]. In addition, in the field of renormalon calculus, one studies the onset of non-perturbative effects from the behaviour near the QCD mass-scale of judiciously selected infinite sub-sets of the perturbative series [2]. Similarly, the extension of the Breit-Wigner formalism to the electro-weak sector of the Standard Model necessitates a non-trivial rearrangement of the perturbative expansion [3]; an analogous task must be undertaken when studying various aspects of finite temperature QCD [4], as well as mass generation, both in 3-dimensional field-theories [5] and in QCD [6], as a prelude to a systematic truncation scheme for the Schwinger-Dyson series.

One of the main difficulties encountered when dealing with the problems mentioned above is the fact that several physical properties, which are automatically preserved in fixed-order perturbative calculations by virtue of powerful field-theoretical principles, may be easily compromised when rearrangements of the perturbative series, such as re-summations, are carried out. These complications may in turn be traced down to the fact that in non-Abelian gauge theories individual off-shell Green's functions (n -point functions) are in general unphysical.

It turns out that this last problem can be circumvented by resorting to the method known as the pinch technique (PT) [6,7]. The PT reorganises systematically a given physical amplitude into sub-amplitudes, which have the same kinematic properties as conventional n -point functions, (propagators, vertices, boxes) [8], but, in addition, are endowed with desirable physical properties. Most importantly, at one-loop order (i) are independent of the gauge-fixing parameter; (ii) satisfy naive, (ghost-free) tree-level Ward identities, instead of the usual Slavnov-Taylor identities. (iii) contain only physical thresholds and satisfy very special unitarity relations [3,9] (iv) coincide with the conventional n -point functions when the latter are computed in the background field method Feynman gauge (BFMFG) [10]. These properties are realized *diagrammatically* by exploiting the elementary Ward identities of the theory in order to enforce crucial cancellations [11], and make manifest intrinsic properties of the S -matrix, which are usually concealed by the quantization procedure.

The important question which arises is whether the PT algorithm may be extended beyond one-loop, leading to the systematic replication of the aforementioned special properties of the PT effective n -point functions to higher orders [12]. In this Letter we will show that the PT can be generalized to two loops by resorting *exactly* to the same physical and field-theoretical principles as at one-loop.

We start by briefly reviewing the one-loop case. Consider the S -matrix element for the quark (u)-antiquark (\bar{u}) scattering process $u(P)\bar{u}(P') \rightarrow u(Q)\bar{u}(Q')$ in QCD; we set $q = P' - P = Q' - Q$, and $s = q^2$ is the square of the momentum transfer. It is convenient to work in the renormalizable Feynman gauge (RFG); this constitutes no loss of generality, since the full S -matrix is independent of the gauge-fixing parameter and gauge-fixing scheme. One first decomposes the elementary tree-gluon vertex $\Gamma_{\alpha\mu\nu}^{(0)}(q, p_1, p_2)$ as follows [7]:

$$\begin{aligned}\Gamma_{\alpha\mu\nu}^{(0)}(q, p_1, p_2) &= [(p_1 - p_2)_\alpha g_{\mu\nu} + 2q_\nu g_{\alpha\mu} - 2q_\mu g_{\alpha\nu}] + [p_{2\nu} g_{\alpha\mu} - p_{1\mu} g_{\alpha\nu}] \\ &= \Gamma_{F\alpha\mu\nu}^{(0)}(q, p_1, p_2) + \Gamma_{P\alpha\mu\nu}^{(0)}(q, p_1, p_2) .\end{aligned}\tag{1}$$

This decomposition assigns a special role to the q -leg, and allows $\Gamma_{F\alpha\mu\nu}^{(0)}$ to satisfy the Ward identity

$$q^\alpha \Gamma_{F\alpha\mu\nu}^{(0)}(q, p_1, p_2) = (p_2^2 - p_1^2) g_{\mu\nu} \quad (2)$$

where the right-hand-side is the difference of two-inverse propagators in the Feynman gauge, and vanishes on shell, i.e. $p_1^2 = p_2^2 = 0$. Notice that the first term in $\Gamma_{F\alpha\mu\nu}^{(0)}$ is a convective vertex, whereas the other two terms originate from gluon spin or magnetic moment. $\Gamma_{F\alpha\mu\nu}^{(0)}(q, p_1, p_2)$ coincides with the BFMFG bare vertex involving one background (q) and two quantum (p_1, p_2) gluons [10].

We then carry out the above decomposition on the three-gluon vertex appearing inside the non-Abelian graph contributing to the one-loop quark-gluon vertex [7]. The result of this is two-fold: First, the action of the longitudinal momenta $p_1^\mu = -k^\mu$, $p_2^\nu = (k - q)^\nu$ on the bare quark-gluon vertices $\Gamma_\mu^{(0)}$ and $\Gamma_\nu^{(0)}$, respectively, triggers the elementary Ward identity of the form $\not{k} = (\not{k} + \not{Q} - m) - (\not{Q} - m)$. The first term gives rise to the pinch contribution $V_{P\alpha\sigma}^{(1)}(q)$ given by $V_{P\alpha\sigma}^{(1)}(q) = 2g^2 C_A \int [dk] [k^2(k+q)^2]^{-1} g_{\alpha\sigma}$, where g is the gauge coupling, C_A is the Casimir eigenvalue of the adjoint representation, and $[dk] = \mu^{2\epsilon} \frac{d^d k}{(2\pi)^d}$, with μ the 't Hooft mass; the second term vanishes on-shell. Second, the part of the graph containing $\Gamma_{F\alpha\mu\nu}^{(0)}$ together with its Abelian-like counterpart defines the PT one-loop quark-gluon vertex $\hat{\Gamma}_\alpha^{(1)}(Q, Q')$, which satisfies the QED-like Ward identity $q^\alpha \hat{\Gamma}_\alpha^{(1)}(Q, Q') = \hat{\Sigma}^{(1)}(Q) - \hat{\Sigma}^{(1)}(Q')$, where $\hat{\Sigma}^{(1)}$ is the PT one-loop quark self-energy. The propagator-like parts extracted from the vertex are cast into the form of a genuine self-energy by setting $\Pi_{P\alpha\beta}^{(1)}(q) = V_P^{(1)}(q)_{\alpha\sigma} t_\beta^\sigma(q)$, where $t_{\mu\nu}(q) = q^2 g_{\mu\nu} - q_\mu q_\nu$; thus, the resulting one-loop PT self-energy reads $\hat{\Pi}_{\alpha\beta}^{(1)}(q) = \Pi_{\alpha\beta}^{(1)}(q) + \Pi_{P\alpha\beta}^{(1)}(q)$. Carrying out the one-loop integrations one finds [6] that the prefactor in front of the logarithm of $\hat{\Pi}_{\alpha\beta}^{(1)}(q)$ is $(11/3)C_A$, i.e. the coefficient of the one-loop β function for quark-less QCD.

For the two-loop case, one considers the two-loop S -matrix element for the aforementioned process $u\bar{u} \rightarrow u\bar{u}$ in the RFG, and focusses on the two-loop quark-gluon vertex $\Gamma_\alpha^{(2)}(Q, Q')$. The Feynman graphs contributing to $\Gamma_\alpha^{(2)}(Q, Q')$ can be classified into two sets. (a) those containing an “external” three-gluon vertex i.e. a three-gluon vertex where the momentum q is incoming (Fig.1). (b) those which do not have an “external” three-gluon vertex. This latter set contains either graphs with no three gluon vertices (abelian-like), or graphs with three-gluon vertices whose all three legs are irrigated by virtual momenta, i.e. q never enters alone into any of the legs. Carrying out the decomposition of Eq. (1) to the external three-gluon vertex of all graphs belonging to set (a), leaving *all* their other vertices unchanged [13], the following situation emerges:

$$\Gamma_\alpha^{(2)}(Q, Q') = \hat{\Gamma}_\alpha^{(2)}(Q, Q') + \frac{1}{2} V_{P\alpha}^{(2)\sigma}(q) \Gamma_\sigma^{(0)} + \frac{1}{2} \Pi_{P\alpha}^{(1)\beta}(q) \left(\frac{-i}{q^2}\right) \hat{\Gamma}_\beta^{(1)}(Q, Q'), \quad (3)$$

with

$$\begin{aligned} V_{P\alpha\sigma}^{(2)}(q) = & -I_1 \left[k_\sigma g_{\alpha\rho} + \Gamma_{\rho\sigma\alpha}^{(0)}(-k, -\ell, k + \ell) \right] (\ell - q)^\rho + (2I_2 + I_3) g_{\alpha\sigma} \\ & I_4 [\Gamma_{\alpha\lambda\rho}^{(0)}(\ell, k, -k - \ell) \Gamma_\sigma^{(0)\lambda\rho}(\ell, k, -k - \ell) - 2k_\alpha(k + \ell)_\sigma], \end{aligned} \quad (4)$$

where $I_1 = I_0(k + \ell)^{-2}(k + \ell - q)^{-2}$, $I_2 = I_0(k + q)^{-2}$, $I_3 = I_0(k + \ell)^{-2}$, $I_4 = I_0\ell^{-2}(k + \ell)^{-2}$, with $iI_0 = g^4 C_A^2 [\ell^2(\ell - q)^2 k^2]^{-1}$, and the two-loop integration prefactor $(\mu^{2\epsilon})^2 \int \int \frac{d^d k}{(2\pi)^d} \frac{d^d \ell}{(2\pi)^d}$ has been suppressed. $\hat{\Gamma}_\alpha^{(2)}(Q, Q')$ is the two-loop BFMFG quark-gluon vertex, $V_{P\alpha\sigma}^{(2)}(q)$ the propagator-like part, and the third term on the right-hand side is the necessary contribution for converting the one-particle reducible part of the two-loop S -matrix element $\Gamma_\alpha^{(0)}(\frac{-i}{q^2}) \Pi_{\alpha\beta}^{(1)}(q) (\frac{-i}{q^2}) \Gamma_\beta^{(1)}(Q, Q')$ into $\Gamma_\alpha^{(0)}(\frac{-i}{q^2}) \hat{\Pi}_{\alpha\beta}^{(1)}(q) (\frac{-i}{q^2}) \hat{\Gamma}_\beta^{(1)}(Q, Q')$. Eq.(3) is a non-trivial result, since there is no a-priori reason why the implementation of the decomposition of Eq. (1) should only give rise to terms which can be interpreted in the way described above. In fact, individual diagrams, or even natural sub-sets of diagrams such as the one-loop three-gluon vertex nested inside the two-loop quark-gluon vertex, give in general rise to contributions which do *not* belong to any of the terms on the right-hand side of Eq. (3). It is only after all terms have been considered that the aforementioned crucial cancellations become possible. Finally, the counterterms of $\Gamma_\alpha^{(2)}(Q, Q')$ must be correctly accounted for [14]. $\hat{\Gamma}_\alpha^{(2)}(Q, Q')$ satisfies the QED-like Ward identity $q^\alpha \hat{\Gamma}_\alpha^{(2)}(Q, Q') = \hat{\Sigma}^{(2)}(Q) - \hat{\Sigma}^{(2)}(Q')$, where $\hat{\Sigma}^{(2)}$ is the two-loop PT quark-self-energy. $\hat{\Sigma}^{(2)}$ is identical to the conventional $\Sigma^{(2)}$ in the RFG (and the BFMFG), exactly as happens at one-loop.

To construct the two-loop PT gluon self-energy $\hat{\Pi}_{\alpha\beta}^{(2)}(q)$, one must append to the conventional two-loop self-energy $\Pi_{\alpha\beta}^{(2)}(q)$ the term $\Pi_{P\alpha\beta}^{(2)}(q) = V_{P\alpha\sigma}^{(2)}(q) t_\beta^\sigma(q)$ together with the term $iR_{P\alpha\beta}^{(2)}(q) = \Pi_{\alpha\beta}^{(1)}(q) V_P^{(1)}(q) + \frac{3}{4} V_{P\alpha}^{(1)\sigma}(q) \Pi_{P\sigma\beta}^{(1)}(q)$

originating from converting a string of two conventional one-loop self-energies into a string of two one-loop PT self-energies [3]. One can show by means of a diagram-by-diagram mapping that the resulting $\widehat{\Pi}_{\alpha\beta}^{(2)}(q)$ is *exactly* identical to the corresponding two-loop self-energy of the BFMFG, and that this correspondence persists after renormalization [14]. Notice that the presence of the term $R_{P\alpha\beta}^{(2)}(q)$ is crucial for the entire construction, and constitutes a non-trivial consistency check of the resummation mechanism first proposed in [3]. An immediate consequence of the above correspondence is that the coefficient in front of the leading logarithm of $\widehat{\Pi}_{\alpha\beta}^{(1)}(q)$ is precisely the coefficient of the two-loop quark-less QCD β function [17], namely $(34/3)C_A^2$ [18]. As a result, one may extend to two-loops the one-loop construction of a renormalization-group-invariant effective charge presented in [16], leading to the unambiguous identification of the conformally-(in)variant subsets of QCD graphs [19]. Finally we note that, exactly as happens at one-loop, the two-loop PT box-graphs are simply the conventional ones in the RFG (and are equal to the ones in the BFMFG).

As has been explained in detail in [3,9], the one-loop PT n -point functions satisfy the optical theorem *individually*. To verify that one starts with the tree-level process $u(P)\bar{u}(P') \rightarrow g(p_1) + g(p_2)$, whose S -matrix element we denote by $\mathcal{T}_{\mu\nu}$; then, one considers the quantity $\mathcal{T}_{\mu\nu}P^{\mu\mu'}(p_1)P^{\nu\nu'}(p_2)\mathcal{T}_{\mu'\nu'}$, where $P_{\mu\nu}(p, \eta) = -g_{\mu\nu} + (\eta_\mu p_\nu + \eta_\nu p_\mu)/\eta p + \eta^2 p_\mu p_\nu / (\eta p)^2$, with η an arbitrary four-vector. One proceeds by first eliminating the $\Gamma_{P\alpha\mu\nu}^{(0)}(q, p_1, p_2)$ part of $\Gamma_{\alpha\mu\nu}^{(0)}(q, p_1, p_2)$, which vanishes when contracted with the term $P^{\mu\mu'}(p_1)P^{\nu\nu'}(p_2)$. Then, the longitudinal parts of the $P_{\mu\mu'}(p_1)$ and $P_{\nu\nu'}(p_2)$ trigger a fundamental cancellation [3,9] involving the s - and t -channel graphs, which is a consequence of the underlying BRS symmetry [15]. Specifically, the action of $p_{1\mu}$ on the $\Gamma_{F\alpha\mu\nu}^{(0)}$ gives

$$p_1^\mu \Gamma_{F\alpha\mu\nu}^{(0)}(q, p_1, p_2) = t_{\alpha\nu}(q) + (p_1^2 - p_2^2)g_{\alpha\nu} + (p_2 - p_1)_\alpha p_{2\nu} ; \quad (5)$$

the first term on the right-hand side cancels against an analogous contribution from the t -channel graph, whereas the second term vanishes for on-shell gluons. Finally, the term proportional to $p_{2\nu}$ is such that (i) all dependence on η vanishes, and (ii) a residual contribution emerges, which must be added to the parts stemming from the $g_{\mu\mu'}g_{\nu\nu'}$ part of the calculation. Then one simply defines self-energy/vertex/box-like sub-amplitudes according to the dependence on $s = (p_1 + p_2)^2$ and $t = (P - p_1)^2$, as in a scalar theory, or QED. The emerging structures correspond to the imaginary parts of the one-loop PT effective Green's functions, as one can readily verify by employing the Cutkosky rules; in fact the residual pieces mentioned at step (ii) above correspond precisely to the Cutkosky cuts of the one-loop ghost diagrams. The one-loop PT structures may be reconstructed directly from this tree-level calculation, without resorting to an intermediate diagrammatic interpretation, by means of appropriately subtracted dispersion relations.

The same procedure must be followed at two-loops; the only difference is that one must now combine contributions from both the one-loop S -matrix element for the process $u(P)\bar{u}(P') \rightarrow g(p_1) + g(p_2)$ and the tree-level S -matrix element for the process $u(P)\bar{u}(P') \rightarrow g(p_1) + g(p_2) + g(p_3)$. The non-trivial point is that the one-loop S -matrix element must be cast into its PT form (as shown in Fig 2a.) *before* any further manipulations take place. Notice that the same procedure which leads to the appearance of $\widehat{\Pi}(q)$ [20] leads also to the conversion of the conventional one-loop three-gluon vertex $\Gamma_{\alpha\mu\nu}^{(1)}(q, p_1, p_2)$ into $\Gamma_{F\alpha\mu\nu}^{(1)}(q, p_1, p_2)$, which is the BFMFG one-loop three-gluon vertex with one background (q) and two quantum (p_1, p_2) [14]. It is straightforward to show that $\Gamma_{F\alpha\mu\nu}^{(1)}(q, p_1, p_2)$ satisfies the following Ward identity

$$q^\alpha \Gamma_{F\alpha\mu\nu}^{(1)}(q, p_1, p_2) = \Pi_{\mu\nu}^{(1)}(p_1) - \Pi_{\mu\nu}^{(1)}(p_2), \quad (6)$$

which is the exact one-loop analogue of the tree-level Ward identity of Eq (2); indeed the right-hand side is the difference of two one-loop self-energies computed in the RFG. In order to extend to the next order the dispersive construction outlined above, one needs the following Ward identity

$$p_1^\mu \Gamma_{F\alpha\mu\nu}^{(1)} = i\widehat{\Pi}_{\alpha\nu}^{(1)}(q) - i\Pi_{\alpha\nu}^{(1)}(p_2) + \lambda_{\nu\sigma}^{(1)}t_\alpha^\sigma(q) + s_\alpha^{(1)}p_{2\nu} \quad (7)$$

with

$$\begin{aligned} \lambda_{\nu\sigma}^{(1)} &= J_3 \left[(k - p_1)^\rho \Gamma_{\nu\rho\sigma}^{(0)}(p_2, k, -k - p_2) - (k + p_2)_\nu k_\sigma \right] - i \left[2B(q) + B(p_1) \right] g_{\nu\sigma} \\ s_\alpha^{(1)} &= J_3 \left[p_2^\sigma k^\rho \Gamma_{F\alpha\sigma\rho}^{(0)}(q, k + p_2, -k + p_1) - p_2 \cdot (k - p_1)(2k + p_2 - p_1)_\alpha \right] + \left(\frac{1}{8} \right) \left[B(p_1) + B(p_2) \right] q_\alpha, \end{aligned} \quad (8)$$

$J \equiv \frac{1}{2}ig^2C_A[k^2(k-p_1)^2(k+p_2)^2]^{-1}$ and $B(p) \equiv g^2C_A \int [dk][k^2(k+q)^2]^{-1}$. Eq. (7) is the one-loop analogue of Eq. (5). The one-loop version of the fundamental BRS-driven cancellation will then be implemented; for instance, the first term on the right-hand side of Eq. (7) will cancel against analogous contributions from the graph of Fig. 2a₂, whereas all remaining terms proportional to $t_{\sigma\alpha}(q)$ will cancel against contributions from the t -channel graphs of Fig. 2a₃.

The same construction must then be repeated for the tree-level process $u\bar{u} \rightarrow ggg$, whose tree-level S -matrix element we denote by $\mathcal{T}_{\mu\nu\rho}$; again, the s -channel graphs (Fig. 2b) must be rewritten in such a way that when contracted with q only terms proportional to p_i^2 emerge, but no transverse pieces, exactly as in Eq.(2). This is accomplished by simply carrying out the decomposition of Eq.(1) only to the vertices where q is entering; then the contributions originating from the $\Gamma_{\rho\alpha\mu\nu}^{(0)}$ parts eventually vanish when contracted with the polarization tensors $P^{\mu\mu'}(p_1)P^{\nu\nu'}(p_2)P^{\rho\rho'}(p_3)$. Acting with the longitudinal parts of the polarization tensors on the $\mathcal{T}_{\mu\nu\rho}$ one must first carry out the corresponding BRS $s-t$ channel cancellation, and pick up *automatically* the correct ghost parts. Notice in particular that this procedure gives rise to the ghost structure given in Fig.3c of [17], which has only three-particle Cutkosky cuts, and does not exist in the conventional formulation.

Adding the s -channel terms together the total propagator-like part emerges; it is proportional to $(34/3)C_A^2q^2$, as it should. Notice that the result is infrared finite, by virtue of crucial cancellations between the the one-loop $u\bar{u} \rightarrow gg$ and the tree-level $u\bar{u} \rightarrow ggg$ cross-sections. The most direct way to verify that is by exploiting the one-to-one correspondence between the terms thusly generated and the Cutkosky cuts of the BFMFG two-loop self-energy; the latter are infrared finite since they effectively originate from a single logarithm.

In conclusion, we have shown that the same physical principles, and, evidently, the same procedure used at one-loop, lead to the generalization of the PT to two-loops. In particular, the known correspondence between PT and BFMFG persists. It would be interesting to explore its origin further, and establish a formal, non-diagrammatic understanding of the PT.

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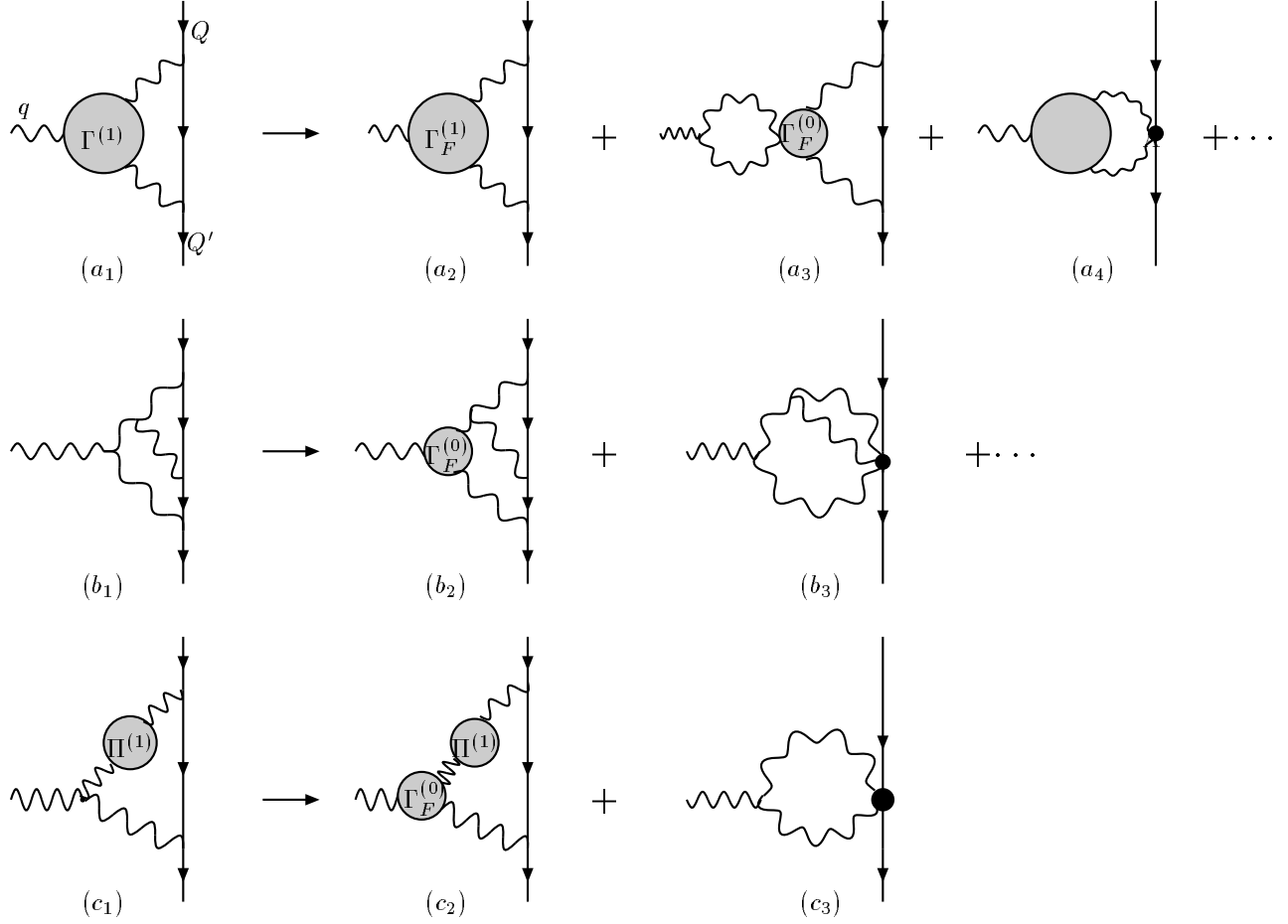


FIG.1 Pinching contributions from two-loop vertex graphs

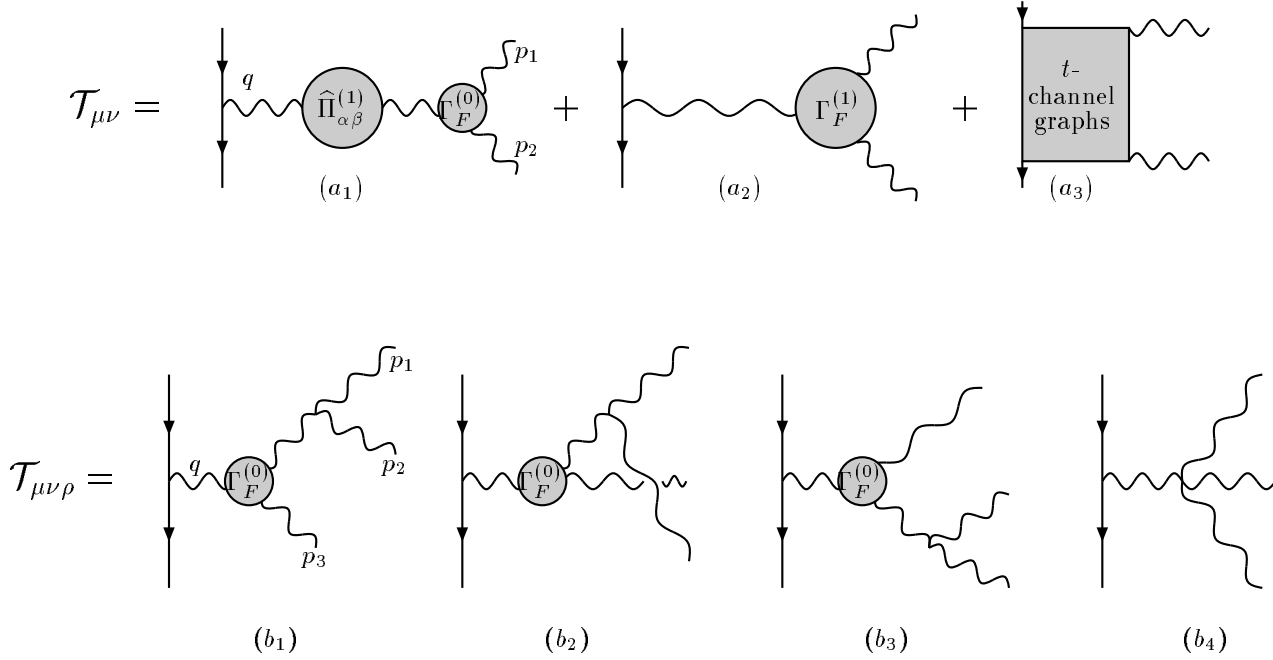


FIG2. The one-loop s -channel corrections to $u\bar{u} \rightarrow gg$, and the tree-level s -channel $u\bar{u} \rightarrow ggg$